D.G.E -HR.SEC. EXAMINATION MARCH - 2016

EXAM ROLL NUMBER 619932

CANDIDATE NAME : ARUN KUMAR T M

UNIQUE ID NO : 1610609932

SUBJECT : 041 MATHEMATICS (ENG)

(C)

APPLIED FOR : SCAN COPY

(B)

D.G.E -HR.SEC. EXAMINATION MARCH - 2016

(FFDBXFIEFAH)

SUB CODE : 041

(FFDBXFIEFAH)

SUBJECT : 041 MATHEMATICS (ENG)

Marks already Awarded	Marks after Retotalling / Revaluation	+/-	Marks in Difference

Designation	Signature
Examiner 1	
Examiner 2	
Examiner 3	

J.D (Ret . / Rev)

J.D (H.S)

Director

Stitching Line

Bundle No: 5 0 8

மதிப்பெண்கள் பக்கம் / Marking Page

Total Marks: 189

அரசுத் தேர்வுகள் துறை
DEPARTMENT OF GOVERNMENT EXAMINATIONS

Packet No. 0 3

HSE

Qn Booklet Series (Tick the appropriate Box)

A	
B	✓

Script No: 0 5

Camp No. 39

விடைத்தாள் திருத்துவோர் நிறைவு செய்ய வேண்டியவை
FOR THE USE OF EXAMINERS ONLY

வினாவாரியாக மொத்தம் Questionwise Total										பக்கவாரியாக மொத்தம் Pagewise Total			
வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	பக்க எண் Page No.	மதிப் பெண்கள் Marks	பக்க எண் Page No.	மதிப் பெண்கள் Marks
1	1	21	1	41	6	61	10	81		1	10	21	6
2	1	22	1	42	6	62	10	82		2	10	22	6
3	1	23	1	43		63		83		3	9	23	0
4	1	24	1	44	6	64	10	84		4	10	24	0
5	1	25	1	45	6	65	6	85		5	10	25	10
6	1	26	1	46	6	66		86		6	5	26	
7	1	27	1	47	6	67		87		7	10	27	
8	1	28	1	48	6	68		88		8	10	28	
9	1	29	1	49		69		89		9	10	29	
10	1	30	1	50		70	10	90		10	6	30	
11	1	31	1	51	6	71		91		11	0	31	
12	1	32	0	52		72		92		12	10	32	
13	0	33	1	53		73		93		13	0	33	
14	1	34	1	54	6	74		94		14	6	34	
15	1	35	1	55	6	75		95		15	6	35	
16	1	36	1	56	10	76		96		16	6	36	
17	1	37	1	57	10	77		97		17	6	37	
18	1	38	1	58	10	78		98		18	6	38	
19	1	39	1	59	10	79		99		19	12	39	
20	1	40	1	60	5	80		100		20	6	40	
மொத்தம் TOTAL	19	மொத்தம் TOTAL	19	மொத்தம் TOTAL	105	மொத்தம் TOTAL	46	மொத்தம் TOTAL		மொத்தம் TOTAL	167	மொத்தம் TOTAL	22

வினாவாரியாக ஒட்டு மொத்தம்
Question-wise Grand Total: 189

பக்கவாரியாக ஒட்டு மொத்தம்
Page-wise Grand Total: 189

AE: 504 14/4/16 FN

SO: 14/4/16 FN 10:054

CE: 14/4/16 FN CE-034

GCP–388-30–57,00,000 Cps.–27-8-15 (HCL-6)–1

HSE

தேர்வு எழுதுபவர் செய்யக்கூடியவை மற்றும் செய்யக்கூடாதவை
Do's & Dont's for Candidates

1. முகப்புச்சீட்டில் உரிய இடத்தில் கையொப்பமிட வேண்டும்.
   Put your signature in the Top sheet in the appropriate place.
2. விடைத்தாளில் ஒரு பக்கத்திற்கு 20 முதல் 25 வரிகள் வரை எழுதவேண்டும்.
   Write 20 to 25 lines in a page.
3. விடைத்தாளின் இருபுறத்திலும் எழுத வேண்டும்.
   Write answers in both sides of paper.
4. செய்முறைகள் யாவும் விடைத்தாளின் பகுதியில் இடம் பெறவேண்டும்.
   All rough works must be done on the lower part of the page.
5. சரியான வினா எண் தவறாமல் எழுத வேண்டும். வினா எண் எழுதாத மற்றும் தவறான வினா எண்கள் குறிப்பிடப்பட்டு எழுதப்பட்ட விடைகள் மதிப்பீடு செய்யப்படமாட்டாது.
   Write the question numbers without fail. Answers without question numbers and wrong question numbers will not be valued.
6. இரு விடைகளுக்கிடையே இடைவெளி விட்டு எழுத வேண்டும்.
   Leave space between two answers.
7. வினாத்தாளின் வரிசை (A or B) மதிப்பெண்கள் பக்கத்தில் குறிக்கப்படல் வேண்டும்
   Question paper booklet series (A or B) should be mentioned in the Marking Page
8. விடைத்தாளில் நீலம்/கருப்புமை கொண்ட பேனாவால் விடைகளை தெளிவாக எழுத வேண்டும்.
   Answers must be legibly written either in Blue or Black ink pen.
9. விடைத்தாளில் எழுதாத பக்கங்களில் குறுக்குக்கோடு இடவேண்டும்.
   Cross the unwritten pages.

1. வினாத்தாளில் எந்தவித குறியீடும் இடக் கூடாது.
   No marking in the question paper.
2. விடைத்தாளை சேதப்படுத்தக் கூடாது.
   Don't damage the answer paper.
3. விடைத்தாளில் எந்த ஒரு பக்கத்திலும் தேர்வு எண்/பெயர் எழுதக்கூடாது.
   Don't write name, Register Number in any page of the answer book.
4. வண்ணக்கலர் கொண்ட பேனா/ பென்சில் எதையும் பயன்படுத்தக் கூடாது.
   Don't write with sketch / colour pencils.
5. விடைத்தாள் கோட்டின் இடது ஓரத்தில் எழுதக்கூடாது.
   Don't write on the left margin.
6. விடைத்தாள் புத்தகத்தின் எந்த தாளையும் கிழிக்கவோ/நீக்கவோ கூடாது.
   Don't tare / remove any page from the answer book.

PART - A

1. 4) fourth quadrant

2. 2) directrix

3. 4) 6

4. 1) -2

5. 1) $8\sqrt{5}\,\lambda$

6. 3) -1

7. 2) $\sqrt{2} - 1$

8. 1) 1

9. ~~4) 1 or $\frac{1}{10}$~~ 2) $\frac{1}{10}$

10. 4) $\frac{dy}{dx} + xy = e^{x}$ is a linear differential Equation is x

10

வினா எண். Qn.No.	
11.	4) $P \wedge (\sim P)$.
12.	1) 4.
13.	2) 2.
14.	1) 5
15.	2) $z_1 + z_3 = z_2 + z_4$.
16.	4) $x = 0$.
17.	1) $f(2a - x) = f(x)$
18.	4) (ii), (iii) (iv)
19.	2) minimum value at $x = 0$.
20.	4) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

16

9

வினா எண். Qn.No.		
21.	1) $\tan^{-1}\left(\frac{2m}{m^2-1}\right)$.	1
22.	4) 5	1
23.	1) 0	1
24.	4) $3\sqrt{30}$.	1
25.	4) 8	1
	(10)	
26.	1) 4	1
27.	4) 9π	1
28.	2) Only a demi-group.	1
29.	4) is a perpendicular bisector of the line joining z_1 and z_2	1
30.	3) 80.	1

19

வினாஎண். Qn.No.	
31. 1	2) trivial Solution and infinitely many non-trivial Solution.
32. 0	2) $x < a$ and $x > -a$.
33. 1	4) $\frac{x e^{ax}}{g(a)}$
34. 1	1) 4.
35. 1	1) $k^3 \det(A)$
36. 1	4) 200 m
37. 1	3) (1, 1, 2)
38. 1	2) $x = \pm \frac{8}{\sqrt{5}}$
39. 1	3) $a = \frac{1}{\lvert m \rvert}$
40. 1	2) $\frac{d^2y}{dx^2} = 0$

9

Part - C.

56.

Given equation:

$$x - 3y - 8z = -10.$$

$$3x + y - 4z = 0.$$

$$2x + 5y + 6z - 13 = 0 \Rightarrow 2x + 5y + 6z = 13$$

$$\begin{pmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 13 \end{pmatrix}$$

$$A \quad X \quad B.$$

The augmented form

$$(A, B) \sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

வினா எண். Qn.No.

57. 10

$\vec{a} = \vec{i} + \vec{j} + \vec{k} = (1, 1, 1)$

$\vec{b} = 2\vec{i} + \vec{k} = (2, 0, 1)$

$\vec{c} = 2\vec{i} + \vec{j} + \vec{k} = (2, 1, 1)$

$\vec{d} = \vec{i} + \vec{j} + 2\vec{k} = (1, 1, 2)$

TO PROVE: $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\,\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\,\vec{d}$

L.H.S

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= \vec{i} + \vec{j} - 2\vec{k}.$$

$$(\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \vec{i} - 3\vec{j} + \vec{k}.$$

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 1 & -3 & 1 \end{vmatrix}$$

$$= -5\vec{i} - 3\vec{j} - 4\vec{k} \longrightarrow (1)$$

R.H.S

$$[\vec{a}\ \vec{b}\ \vec{d}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= -1 - 3 + 2 = -2$$

$$[\vec{a}\ \vec{b}\ \vec{d}]\vec{c} = -2\{2\vec{i} + \vec{j} + \vec{k}\}$$

$$= -4\vec{i} - 2\vec{j} - 2\vec{k}$$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= 0\vec{i} - 1 + 0 + 2 = 1$$

$$[\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = 1\{\vec{i} + \vec{j} + 2\vec{k}\} = \vec{i} + \vec{j} + 2\vec{k}.$$

வினா எண். Qn.No.

$[\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = -4\vec{i} - 2\vec{j} - 2\vec{k} - (\vec{i} + \vec{j} + 2\vec{k})$

$= -5\vec{i} - 3\vec{j} - 4\vec{k}$ ——→ (2)

By (1) & (2)

we have.

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

58. Given line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$

dir $\vec{u} = (2, 3, -2)$
$\quad\quad\quad\quad\ \ \ell \quad m \quad n.$

$\vec{b} = (2, 2, 1)$
$\quad\quad\ \ x_2\ y_2\ z_2$

Given point $\vec{a} = (-1, 1, -1)$.
$\quad\quad\quad\ \ x_1\ y_1\ z_1$

The Reg plane passes through $\vec{a}$, $\vec{b}$ and $\parallel^{el}$ to $\vec{u}$.

வினா எண். Qn.No.

Vector Equation $\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{c}$

$\vec{r} = (1-s)(-\vec{i} + \vec{j} - \vec{k}) + s(2\vec{i} + 2\vec{j} + \vec{k}) + t(2\vec{i} + 3\vec{j} - 2\vec{k})$

Cart Equation :

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0$$

$$\begin{vmatrix} x+1 & y-1 & z+1 \\ -3 & -1 & -2 \\ 2 & 3 & -2 \end{vmatrix} = 0$$

$(x+1)(2+6) - (y-1)(6+4) + (z+1)(-9+2) = 0$

$(x+1)(8) - (y-1)(10) + (z+1)(-7) = 0$

$8x + 8 - 10y + 10 - 7z - 7 = 0$

$8x - 10y - 7z + 11 = 0$

வினா எண். Qn.No.

59. 10

$$\operatorname{Re}\left(\frac{z+1}{z+i}\right) = 1$$

Let $z = x + iy$.

Consider:

$$\frac{z+1}{z+i} = \frac{x+iy+1}{x+iy+i}$$

$$= \frac{(x+1)+iy}{x+i(y+1)}$$

$$= \frac{(x+1)+iy}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$$

$$\frac{z+1}{z+i} = \frac{(x+1)x + y(y+1) + \text{Imp part}}{x^2 + (y+1)^2}$$

Given:

$$\operatorname{Re}\left(\frac{z+1}{z+i}\right) = 1$$

வினா எண், Qn.No.

$$\frac{(x+1)x + y(y+1)}{x^2+(y+1)^2} = 1$$

$$x^2 + x + y^2 + y = x^2 + y^2 + 2y + 1$$

$$\boxed{x - y = 1}$$

is the req locus which is a straight line.

---

60. Given parabola :

$$y^2 + 8x - 6y + 1 = 0.$$

$$y^2 - 6y = -8x - 1$$

$$(y-3)^2 - 9 = -8x - 1$$

$$(y-3)^2 = -8x + 8.$$

$$(y-3)^2 = -8(x+1)$$

$$Y^2 = -8X \quad \text{s.t} \quad Y = y-3$$

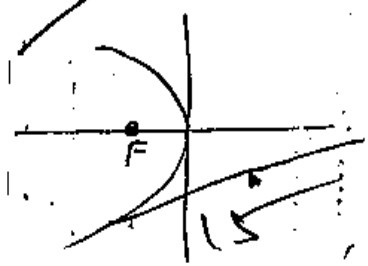
$$X = x+1.$$

{S.F $y^2 = -4ax$}.

Comparing $-4a = -8$.

$$\boxed{a = 2}.$$

Rough.



எடுத்துக்காட்டு எண். Qn.No.

	w.r.t to x, y axes	w.r.t to x, y axes $x = X - 1$ $y = Y + 3$
Vertex	$(0, 0)$	$x = -1$ $y = 3$
Focus	$(a, 0) = (2, 0)$	$x = -3$ $y = 3$
Eqn of directrix	$x = a$ $\boxed{X = 2}$	$\boxed{x = 1}$
Latus rectum	$X = -a$ $\boxed{X = -2}$	$\boxed{x = -3}$
L.L.R.	$4a = 8$ units	8 units
Axis	$\boxed{Y = 0}$	$\boxed{y = 3}$

$(-1, 3)$

$(-3, 3)$

Result:

Axis $y = 3$

Vertex $(-1, 3)$

Focus $(-3, 3)$

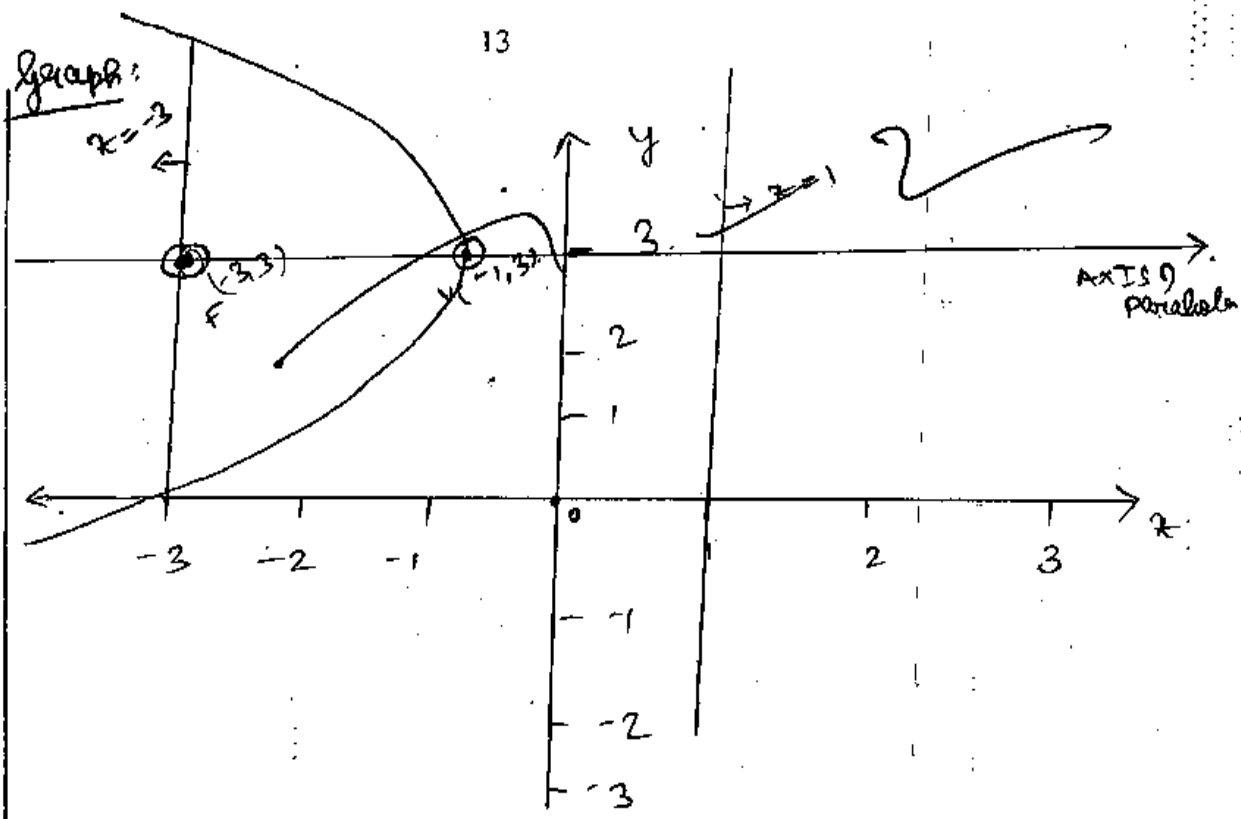
Eqn of directrix $x = 1$

Latus rectum $x = -3$

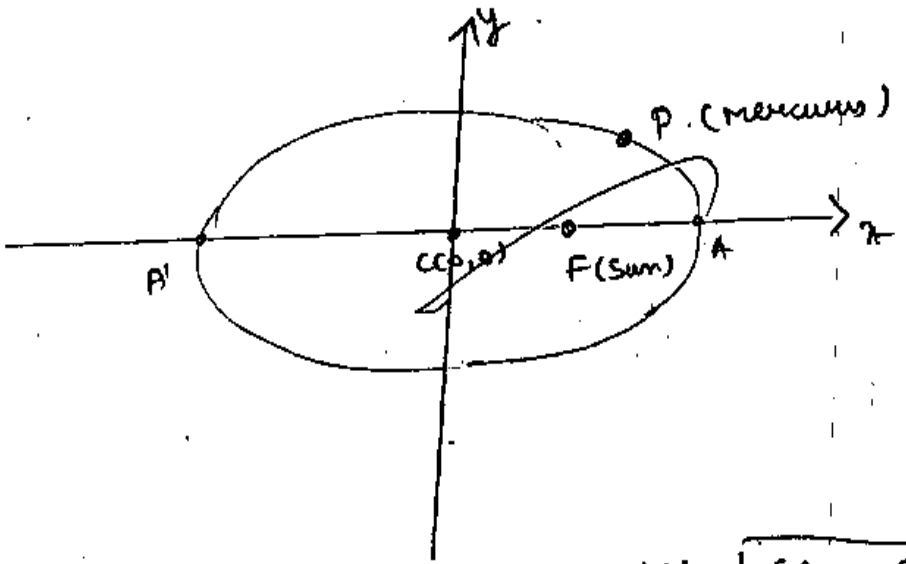
L.L.R $= 8$ units

வினா எண். Qn.No.

Graph:



61. 10



WKT $\boxed{CA = a}$.

Given $a = 36$ million miles

$e = 0.206$.

To find i) $FA = ?$

ii) $F'A = ?$

10

வினா எண். Qn.No.

$CA = CF + FA$.

$FA = CA - CF$.

$= a - ae$

$FA = a(1 - e)$.

$= 36(1 - 0.206)$

$= 36(0.794)$

9.999
0.206
0.794
×36
4764
2382
28.584

$FA = 28.584$ million mile.

$FA' = CF + CA'$

$= a + ae$

$= a(1 + e)$

$= 36(1 + 0.206)$

1.000
206
1.206
×36
7236
3618
43.416

$FA' = 43.416$ million mile

Result:

i) Closest distance mercury come to Sun is 28.584 million mile.

ii) The greatest possible distance between Mercury and Sun is 43.416 million mile

62. Given line $x - y + 4 = 0$

$y = x + 4$

S.F $\{y = mx + c\}$

Comparing $m = 1$

$c = 4$.

Given ellipse $x^2 + 3y^2 = 12$

$\frac{x^2}{12} + \frac{y^2}{4} = 1$

S.F $\left\{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right\}$

$a^2 = 12$ $b^2 = 4$

The condition for the line to be tgt for ellipse

$c^2 = a^2 m^2 + b^2$.

$16 = 12(1) + 4$

$16 = 16$ is true

$\Rightarrow$ The Given line is the tgt to the ellipse.

10

வினாஎண்.
எண்ணர்.
Qn.No.

Point of Contact : $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$

$= \left(\frac{-12(1)}{4}, \frac{4}{4}\right)$

$= (-3, 1)$

Result:

The point of contact is $(-3, 1)$.

---

64. (10)

Given $u = \sin 3x \cos 4y$.

To verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

வினா எண். Qn.No.

L.H.S

$$\frac{\partial u}{\partial x} = 3\cos 3x \cos 4y.$$

$$\frac{\partial^2 u}{\partial x \partial y} = -12 \cos 3x \sin 4y. \longrightarrow (1)$$

R.H.S

$$\frac{\partial u}{\partial y} = -4 \sin 3x \sin 4y$$

$$\frac{\partial^2 u}{\partial y \partial x} = -12 \cos 3x \sin 4y \longrightarrow (2)$$

By (1) & (2) we have.

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$

வினா எண். Qn.No.

65. Given

$$x = a(t + \sin t)$$

$$y = a(1 + \cos t) \qquad 0 \le t \le 2\pi.$$

$$S.A = 2\pi \int_a^b y \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt.$$

Let us take the limits 0 to π.

$$a = 0 \qquad b = \pi.$$

$$\frac{dx}{dt} = a(1 + \cos t)$$

$$\frac{dy}{dt} = a(-\sin t)$$

$$\left(\frac{dx}{dt}\right)^2 = a^2(1 + \cos t)^2$$

$$= a^2(1 + 2\cos t + \cos^2 t)$$

$$\left(\frac{dy}{dt}\right)^2 = a^2 \sin^2 t.$$

வினா எண். Qn.No.

$$S.A = 2\pi \int_0^{\pi} y \sqrt{a^2(1+2\cos t(\cos^2 t) + a^2 \sin^2 t}\, dt$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2(1+2\cos t + \cos^2 t + \sin^2 t)}\, dt \quad (\sin^2 t + \cos^2 t = 1)$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2(2+2\cos t)}\, dt$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2 2(1+\cos t)}\, dt$$

$$= 2\pi \int_0^{\pi} y \cdot \sqrt{a^2 2(2\cos^2 t/2)}\, dt$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2 4 \cos^2 t/2}\, dt$$

$$= 2\pi \int_0^{\pi} y \left[2a \cos t/2\right] dt$$

$$= 2\pi \int_0^{\pi} a(1+\cos t)\left[2a \cos t/2\right] dt$$

$$= 4\pi a^2 \int_0^{\pi} \cos^2 t/2 \cos t/2\, dt$$

$$= 4\pi a^2 \int_0^{\pi} \cos^3 t/2\, dt$$

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$$= 4\pi a^2 \int_0^{\pi} \cos^3 t/2 \; dt$$

$t/2 = x$

$\frac{dt}{dx} = 2$

$dt = 2dx$

t	0	π
x	0	$\pi/2$

$$= 4\pi a^2 \; 2 \int_0^{\pi/2} \cos^3 x \; dx$$

$$= 8\pi a^2 \left[\frac{2}{3} \cdot 1\right]$$

$$= \frac{16\pi a^2}{3} \text{ Sq. unit}$$

By Reduction Formula:

$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \frac{n-2}{n-3} \cdots \frac{2}{3}$ n is odd.

The Rqd Surface area $= \frac{16\pi a^2}{3}$.

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63.

$x = a(\theta + \sin\theta)$ $\quad y = a(1+\cos\theta)$.

$\theta = \pi/2$.

$$\frac{dx}{d\theta} = a(1+\cos\theta) = a\{2\cos^2\theta/2\}$$

$$\frac{dy}{d\theta} = a(-\sin\theta) = -a\{2\sin\theta/2\cos\theta/2\}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \cancel{a(-\sin\theta)}$$

$$= -\cancel{2}\{\cancel{2}\sin\theta/2 \times \cos\theta/2\} \times \frac{1}{\cancel{2}\times\cancel{2}\times\cos^2\theta/2}$$

$$= \frac{-\sin\theta/2}{\cos\theta/2}$$

$$\boxed{\frac{dy}{dx} = -\tan\theta/2} \Rightarrow m.$$

அதிலான

at $\theta = \pi/2$ $\quad x = a(\pi/2 + 1)$ அதேபோல்

$y = a($

The coordinate is (x, y) is $\left(a\frac{\pi}{2}+a, a\right)$

Eqn of tgt · at $(a\pi/2 + a, a)$

$$(y - y_1) = m(x - x_1)$$

$$(y - a) = -\tan\theta/2 \{x - (a\pi/2 + a)\}$$

$$y - a = -\tan\theta/2 \left(x - \frac{a\pi}{2} + a\right)$$

Slope of normal $m_2 = \frac{-1}{m_1} = +\cot\theta/2$

Eqn of normal at $(a\pi/2 + a, a)$

$$(y - y_1) = \frac{-1}{m}(x - x_1)$$

$$(y - a) = \cot\theta/2 \left(x - \frac{a\pi}{2} - a\right).$$

To 10

a) Given curve $y = e^{-x^2}$

y' = $e^{-x^2} \cdot -2x$

$-2y + -2x\,y'$

$y'' = -2e^{-x^2} + (+4x^2 e^{-x^2})$

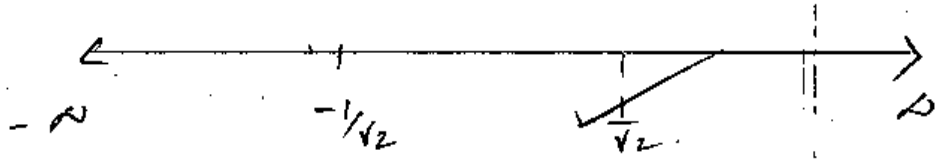
Put $y'' = 0$

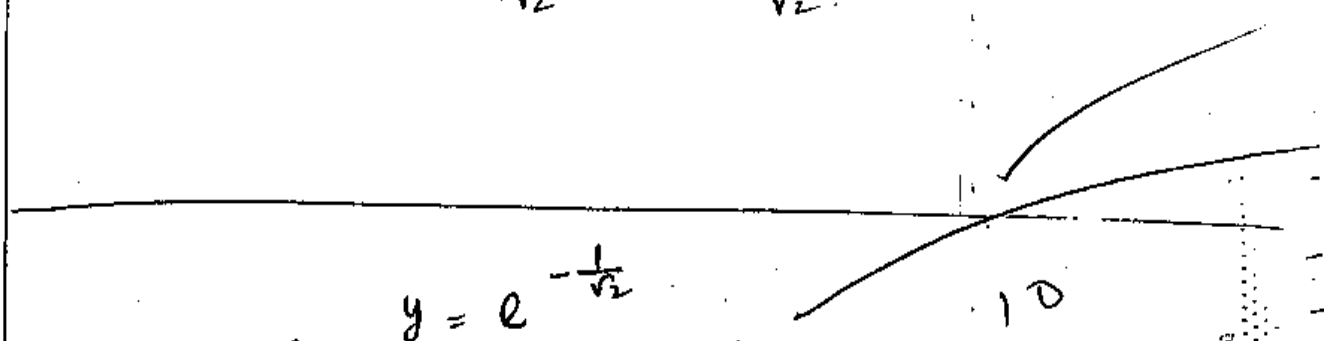
$-2e^{-x^2} + 4x^2 e^{-x^2} = 0$

$+2xe^{-x^2} = +4x^2 e^{-x^2}$

$x^2 = \frac{1}{2}$

$\boxed{x = \pm \frac{1}{\sqrt{2}}}$





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Interval	Sign of y''	Concavity
$-\infty < x < -1/\sqrt{2}$ Say (-1).	+ve	Concave upward.
$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ Say $x=0$	−ve	Concave downward
$\frac{1}{\sqrt{2}} < x < \infty$ Say $x=$	+ve	Concave upward.

(i) The sign of y'' changes from +ve to to −ve at $x = -\frac{1}{\sqrt{2}}$,

$\left(\frac{-1}{\sqrt{2}}, f(x)\right)$ is the point of inflexion

$\left(-\frac{1}{\sqrt{2}}, e^{-1/2}\right)$ is the point of inflexion

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② The sign of f'' changes from -ve to +ve at $x = \frac{1}{\sqrt{2}}$.

$\left(\frac{1}{\sqrt{2}}, f(x)\right)$ is the pt of inflection

$\left(\frac{1}{\sqrt{2}}, e^{-1/2}\right)$ is the pt of inflection

Result :

1. $\left(-\infty < x < -\frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{2}} < x < \infty\right)$ → Concave upward

2. $\left(\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}\right)$ → concave downward

3. $\left(\frac{-1}{\sqrt{2}}, e^{-1/2}\right)$ $\left(\frac{1}{\sqrt{2}}, e^{-1/2}\right)$ is the pt of inflection.

வினா எண். Qn.No.

41.

Given Equation.

$$x + y + 2z = 4$$

$$2x + 2y + 4z = 8$$

$$3x + 3y + 6z = 10.$$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{vmatrix} = 0 \quad \{ R_1 \equiv R_2 \}$$

$\Rightarrow$ Solution may are may not exist

$$\Delta_x = \begin{vmatrix} 4 & 1 & 2 \\ 8 & 2 & 4 \\ 10 & 3 & 6 \end{vmatrix} = 0 \quad \{ R_2 \propto R_3 \}$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ 3 & 10 & 6 \end{vmatrix} = 0 \quad \{ R_1 \propto R_3 \}$$

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$$\Delta z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 2 & 8 \\ 3 & 3 & 10 \end{vmatrix} = 0 \quad \{R_1 \equiv R_2\}$$

$\Delta = 0$ and $\Delta x = \Delta y = \Delta z = 0$

$\Rightarrow$ Consistent and infinitely many solution

Let consider 2×2 minor of Δ.

It is found that all the 2×2 minor of Δ has $\Delta = 0$

But one of the 2×2 Minor of $\Delta z \neq 0$

$$\begin{vmatrix} 8 & 2 \\ 10 & 3 \end{vmatrix} = 24 - 20 = 4 \neq 0.$$

Since, on taking 2×2 minor, we have $\Delta = 0$ But $\Delta z \neq 0$..

$\Rightarrow$ no solution exist.

b

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42. Let $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ be a Given matrix ——→ (1)

$$A_C = \begin{pmatrix} +(-4) & -(-1) & +4 \\ -(+3) & +0 & -(-4) \\ +(-3) & -(-1) & +3 \end{pmatrix}$$

Co-fact of -4 is $\begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = -4$

Co fact of -3 is $\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = -1$

Co fact of -3 is $\begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4$

Co-factor of 1 is $\begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = +3$

வினா எண். Qn.No.

Co-factor of 0 is $\begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix}$

$= -12 + 12 = 0$.

Co-factor of 1 is $= \begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix}$.

$= -16 + 12$

$= -4$

Cofactor of 4 is $\begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix}$

$= -3$

Cofactor of 4 is $\begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix}$

$= -4 + 3 = -1$

Cofactor of 3 is $= \begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix} = +3$

வினா எண்.
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$$A_C = \begin{pmatrix} -4 & +1 & 4 \\ -3 & 0 & 4 \\ -3 & +1 & 3 \end{pmatrix}$$

$$A_C^T = \begin{pmatrix} 4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{pmatrix} \longrightarrow (2)$$

$= A$ itself $\longrightarrow$

Therefore By (1) & (2) we have adjoint A of the given matrix is A itself.

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44 i)

$\vec{F} = (2, -2, 1)$

$5\vec{F} = \dfrac{5(2, -2, 1)}{\sqrt{4+4+1}}$

$= \dfrac{5(2, -2, 1)}{\sqrt{9}}$

$= \dfrac{5}{3}(2, -2, 1)$

$d =$ to - From

$= (5, 3, 7) - (1, 2, 3)$

$d = (4, 1, 4)$

$W = \vec{F} \cdot \vec{d}$

$= 5/3 (2, -2, 1) \cdot (4, 1, 4)$

$= 5/3 (8 - 2 + 4)$

$= 5/3 (10)$

$= 50/3$ units

6

6

ii). $A = (-1, 4, -3)$.

Sphere: $x^2+y^2+z^2-3x-2y+2z-15=0$.

G. F. $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

On Comparing.

$2u = -3$ | $2u = -2$ | $2w = 2$

$u = -3/2$ | $u = -1$ | $w = 1$

$d = -15$.

Centre $= (-u, -v, -w)$

$= (3/2, 1, -1)$

Let (x, y, z) be the point B)

By Mid Point Formula.

$\left(\frac{-1+x}{2}, \frac{4+y}{2}, \frac{-3+z}{2}\right) = (3/2, 1, -1)$

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Comparing

$\frac{-1+x}{2} = \frac{3}{2}$ | $\frac{4+y}{2} = 1$ | $\frac{-3+z}{2} = -1$

$x = 4$ | $4+y = 2$ | $-3+z = -2$

| $y = -2$ | $z = +1$

The req'd point B is $(4, -2, +1)$.

---

45. b

Given equation $x^4 - 4x^2 + 8x + 35 = 0$

Given root $= 2 + \sqrt{3}i$

Other root $= 2 - \sqrt{3}i$ { Complex roots occur in conjugate pairs }

Sum of Roots $= 4$

Product of Roots $= 2^2 + (\sqrt{3})^2 = 4 + 3 = 7$

Corresponding factor $= x^2 - 4x + 7$

To find the other factor

$(x^4 - 4x^2 + 8x + 35) = (x^2 - 4x + 7)(x^2 + px + 5)$

On Comparing x

$8 = 7p - 20$

$7p = 28$ $\quad p = 4$

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The ② find other 4 roots.

$$z^2 + 4z + 5 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Here $a = 1$, $b = 4$, $c = 5$ 4.1.

$$z = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$z = \frac{-4 \pm \sqrt{-4}}{2}$$

$$z = \frac{-4 \pm i2}{2}$$

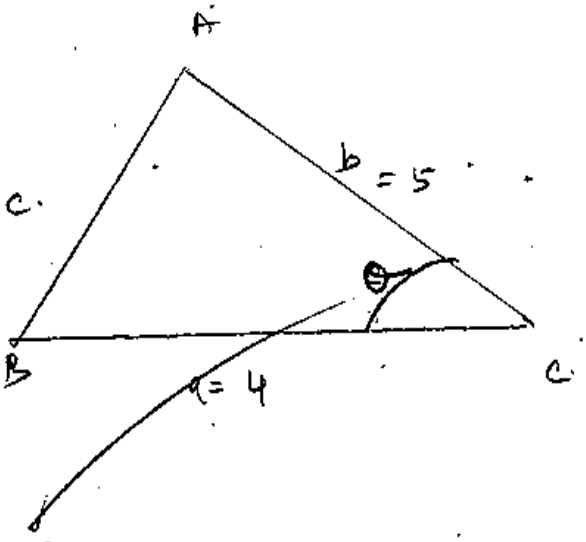
$$z = \frac{2(-2 \pm i)}{2}$$

$$z = -2 \pm i$$

The roots are

$$2 \pm \sqrt{3}i, \quad -2 \pm i.$$

47.



Given: $a = 4$ m

$b = 5$ m.

$$\frac{d\theta}{dt} = 0.06 \text{ rad/sec}$$

To find $\left.\frac{dA}{dt}\right]_{\theta = \pi/3}$ = ?

$$\text{Area} = \frac{1}{2} ab \sin\theta$$

$$\frac{dA}{dt} = \frac{1}{2} ab \cos\theta \cdot \frac{d\theta}{dt}$$

$$= \frac{1}{2} \times 4 \times 5 \times \cos\frac{\pi}{3} \times 0.06$$

$$= 10 \times \frac{1}{2} \times 0.06$$

$$= 0.3 \text{ m}^2/\text{sec}$$

6

The area of the triangle is increasing with rate of $0.3 \text{ m}^2/\text{sec}$.

வினா எண். Qn.No.

46. To prove $\tan^{-1}(x) < x \quad \forall\ x > 0$

$f(x) = x - \tan^{-1}(x)$

$f'(x) = 1 - \frac{1}{1+x^2} > 0 \quad \forall\ x > 0$

$f(x)$ is strictly increasing on $\forall\ x > 0$

wkt. If $x > 0$.

$f(x) > f(0)$

$x - \tan^{-1}(x) > 0 - \tan(0)$

$x - \tan^{-1}(x) > 0$

$x > \tan^{-1} x$

$\tan^{-1} x < x \quad \forall\ x > 0$

Hence proved.

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48

b

Given $V = Ze^{ax+by}$ → (1)

Given Z is a homogeneous function of degree n in x & y.

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz \quad \rightarrow (2).$$

On partially diff (1). wrt x.

$$\frac{\partial V}{\partial x} = Ze^{ax+by}$$

$$\frac{\partial V}{\partial x} = Ze^{ax+by}\,a + e^{ax+by}\frac{\partial z}{\partial x}.$$

$$x\frac{\partial V}{\partial x} = axZe^{ax+by} + xe^{ax+by}\frac{\partial z}{\partial x}.$$

On partial diff (1) wrt y.

$$\frac{\partial V}{\partial y} = Ze^{ax+by}\cdot b + e^{ax+by}\frac{\partial z}{\partial y}.$$

$$y\frac{\partial V}{\partial y} = byZe^{ax+by} + ye^{ax+by}\frac{\partial z}{\partial y}.$$

12

வினா எண். Qn.No.

$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}$ -

$= e^{ax+by}\left[ax Z + x\frac{\partial Z}{\partial x}\right] +$

$e^{ax+by}\left[by Z + y\frac{\partial Z}{\partial x}\right]$.

$= e^{ax+by}\left[ax Z + bxyz + x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial x}\right.$

by (2).

$= e^{ax+by}\left[axz + bxyz + nZ\right]$

$= Z e^{ax+by}\left[ax + by + n\right]$

$= V(ax+by+n)$

Hence proved.

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51, b

TO prove: $[(\sim q) \wedge p] \wedge q$ is a contradiction

p	q	$(\sim q)$	$[\sim q \wedge p]$	$((\sim q) \wedge p) \wedge q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

↓ ①

Since all values of Last column are F.

From ① it is clear that

$[(\sim q) \wedge p] \wedge q$ is a contradiction

b

53. ~~$n = 2$~~

54. ~~$p = \frac{20}{100}$~~ =

54.

$n = 10$

x be r.v denoting choosing defective bol

$P(\text{of choosing defective}) = \frac{20}{100} = \frac{1}{5} = p$

$P(\text{of choosing non defective}) = \frac{4}{5} = q$

B. Exp.

$P(X = x) = n C_x \, p^x q^{n-x}$

$P(X=2) = 10 C_2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^8$

$= \frac{10 \cdot 8}{1 \cdot 2} \frac{16}{5^2} \left(\frac{1}{5^8}\right)$.

$= \frac{640}{5^{10}}$

அண்ணாவ் அடுத்தப்பட

(ii) poisson :

$\lambda = np \qquad 10 \times \frac{4}{5} = 8$

$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$ ... ...

54.

b

$n = 10$.

X be r.v denoting choosing a defective bolt

$x = 0, 1, 2$.

$P(\text{choosing defective bolt}) = \frac{20}{100} = \frac{1}{5} = p$

$P(\text{choosing non-defective bolt}) = \frac{80}{100} = \frac{4}{5} = q$

i) Binomial $= P(X=x) = {}^nC_x \, p^x q^{n-x}$

$P(X=2) = 10C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8$

$= \frac{10^5 \cdot 9}{1 \cdot 2} \left(\frac{4^8}{5^{10}}\right)$

$= 45 \times \frac{4^8}{5^{10}}$

6

Poisson : $P(x=2)$

$np = p \times \frac{1}{5} = 2 = \lambda$

$$P(x=2) = \frac{e^{-2} 2^2}{2!}$$

$$= \frac{0.1353 \times 4}{1 \cdot 2}$$

$$= 0.2706$$

$e^{-\lambda} \lambda^x / x!$

$0.1353 \times 2 = 706$

Result:

Binomial : $\frac{45 \times 4^5}{5^{10}}$

Poisson = 0.2706.

55 b) (i) Given $\left(\frac{1+i}{1-i}\right)^n = 1$.

Consider : $= \frac{1+i}{1-i^2}$

$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)^2}{2}$

$= \frac{1 + 2i - 1}{2}$

$= \frac{2i}{2}$

$\left(\frac{1+i}{1-i}\right)^n = i^n = 1$

$i^{4, 8, 12 \ldots} = 1$

Least positive integer $n = 4$

(ii) $(i)^{1/3}$

Consider i° : $\cos \pi/2 + i \sin \pi/2$.

$$(i)^{1/3} = \left(\cos \pi/2 + i \sin \pi/2\right)^{1/3}$$

$$= \left[\cos\left(2k\pi + \pi/2\right) + i \sin\left(2k\pi + \pi/2\right)\right]^{1/3}$$

$k = 0, 1, 2$

By De moivre

$$= \left[\cos \frac{1}{3}(2k\pi + \pi/2) + i \sin 1/3(2k\pi + \pi/2)\right]$$

when $k = 0 \Rightarrow$ cis $\pi/6$.

$k = 1 \Rightarrow$ cis $\frac{5\pi}{6}$.

$k = 2 \Rightarrow$ cis $9\pi/6$.

$2\pi + \pi/2$

$\frac{5\pi}{2}$

$4\pi + \pi/2$

$\frac{9\pi}{2}$

The values are cis $\pi/6$, cis $5\pi/6$, cis $9\pi/6$

53.

$n = 2$

x be r.v denoting getting Aces.

$x = 0, 1, 2$.

$P(S) = P(\text{Probability of getting Aces}) = \frac{4}{52}$

$P(F) = \frac{48}{52}$.

$P(x=0) = P(FF)$

$= \frac{\cancel{48}^{12}}{\cancel{52}_{13}} \times \frac{\cancel{48}^{12}}{\cancel{52}_{13}}$

$= \frac{144}{169}$.

$P(x=1) = P(SF \text{ or } FS)$

$= P(SF) \times 2$

$= \frac{\cancel{48}^{12}}{\cancel{52}_{13}} \frac{\cancel{4}^{1}}{\cancel{52}_{13}} \times 2$

$= \frac{24}{169}$.

$P(x=2) = P(SS)$.

$= \frac{4}{52}\frac{4}{52} = \frac{1}{169}$.

0

Probability mass function :

x	0	1	2	
P(x=x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$	$\sum p_i = 1$

Mean = $\sum p_i x_i$

$= \frac{144}{169}(0) + \frac{24}{169} \times (1) + (2 \times \frac{1}{169}$

$= \frac{24}{169} + \frac{2}{169}$

$\boxed{E(x) = \frac{26}{169}.}$

Var = $E(x^2) - (E(x))^2$

$E(x^2) = \sum p x^2$

$= (0)^2 \times \frac{144}{169} + (1^2) \times \frac{24}{169} + 12^2 \times \frac{1}{169}$

$= \frac{24}{169} + \frac{4}{169} = \frac{28}{169}.$

Var = $\frac{28}{169} - \left(\frac{26}{169}^2\right)^2$

$= \frac{28}{169} - \frac{4}{169} = \frac{24}{169}.$

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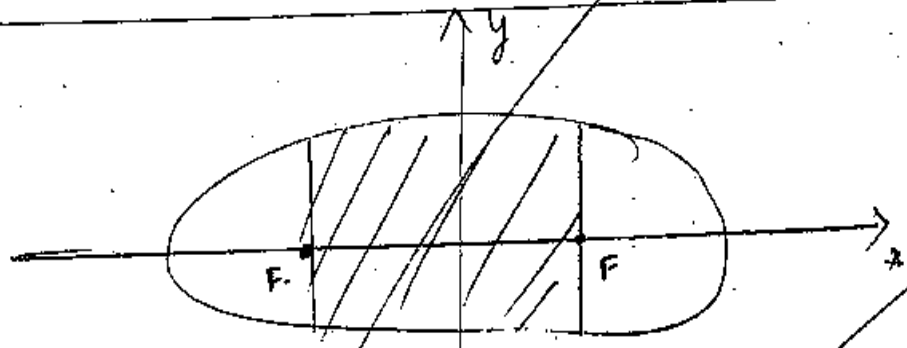
[redacted] .....

Result:

Mean = $\frac{26}{169}$

Variance = $\frac{24}{169}$

66.



The given ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$. (Symmetrical both the Axes) ↳ ①.

(PTO)

56.

$$x - 3y - 8z = -10$$

$$3x + y - 4z = 0$$

$$2x + 5y + 6z = 13$$

$$\begin{pmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 13 \end{pmatrix}$$

Augument form:

$$(A, B) \sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 0 & 0 & 0 \end{pmatrix} R_3 \to \frac{R_3}{11} - \frac{R_2}{10}$$

It is in echelon form.

$P(A,B) = 2 = P(A)$ < No of unknowns (3)

Considered A has unique Solutions.

To find Solution, Let us take $z = k$ $(k \in R)$ k is an arbitrary constant

By backward calculation.

~~$10y + 20z =$~~

$10y + 20z = 30$

$10y = 30 - 20k$

$\boxed{y = 3 - 2k}$

$x - 3y - 8z = -10.$

$x - 3y = -10 + 8k.$

10

$x - 3(3-2k) = -10 + 8k$

$x - 9 + 6k = -10 + 8k$

$x = 2k - 1$

The Solution :

$x = 2k - 1$ $\quad y = 3 - 2k$ $\quad x = 2k - 1$

$y = 3 - 2k$

$z = k \qquad k \in \mathbb{R}$.

The Solution are

$$x = 2k - 1$$
$$y = 3 - 2k$$
$$z = k \qquad \{k \in \mathbb{R}\}$$