

D.G.E -HR.SEC. EXAMINATION MARCH - 2016



EXAM ROLL NUMBER **619932**

CANDIDATE NAME : ARUN KUMAR T M

UNIQUE ID NO : 1610609932

SUBJECT : **041 MATHEMATICS (ENG)**

APPLIED FOR : SCAN COPY



(C)

D.G.E -HR.SEC. EXAMINATION MARCH - 2016

(B)

(FFDBXRIEFAH)



SUBJECT : **041 MATHEMATICS (ENG)**

Marks already Awarded	Marks after Retotalling / Revaluation	+/-	Marks in Difference

Designation	Signature
Examiner 1	
Examiner 2	
Examiner 3	

J.D (Ret. / Rev)

J.D (H.S)

Director

SUB CODE : 041
(FFDBXRIEFAH)



Stitching Line



Bundle No:

மதிப்பெண்கள் பக்கம் / Marking Page

Total Marks

508

Packet No.

03

அரசுத் தேர்வுகள் துறை
DEPARTMENT OF GOVERNMENT EXAMINATIONS

189

Script No:

05

HSE

On Booklet Series
(Tick the appropriate Box)

A	
B	✓

Camp No. 39

விடைத்தாள் திருத்துவோர் நிறைவு செய்ய வேண்டியவை
FOR THE USE OF EXAMINERS ONLY

வினாவாரியாக மொத்தம் Questionwise Total										பக்கவாரியாக மொத்தம் Page-wise Total			
வினா எண் Q.No.	மதிப்பு பெண்கள் Marks	வினா எண் Q.No.	மதிப்பு பெண்கள் Marks	வினா எண் Q.No.	மதிப்பு பெண்கள் Marks	வினா எண் Q.No.	மதிப்பு பெண்கள் Marks	வினா எண் Q.No.	மதிப்பு பெண்கள் Marks	பக்க எண் Page No.	மதிப்பு பெண்கள் Marks	பக்க எண் Page No.	மதிப்பு பெண்கள் Marks
1	1	21	1	41	6	61	10	81		1	10	21	6
2	1	22	1	42	6	62	10	82		2	19	22	6
3	1	23	1	43		63		83		3	9	23	0
4	1	24	1	44	6	64	10	84		4	10	24	0
5	1	25	1	45	6	65	6	85		5	10	25	10
6	1	26	1	46	6	66		86		6	5	26	
7	1	27	1	47	6	67		87		7	10	27	
8	1	28	1	48	6	68		88		8	10	28	
9	1	29	1	49		69		89		9	10	29	
10	1	30	1	50		70	10	90		10	6	30	
11	1	31	1	51	6	71		91		11	0	31	
12	1	32	0	52		72		92		12	10	32	
13	0	33	1	53		73		93		13	0	33	
14	1	34	1	54	6	74		94		14	6	34	
15	1	35	1	55	6	75		95		15	6	35	
16	1	36	1	56	10	76		96		16	6	36	
17	1	37	1	57	10	77		97		17	6	37	
18	1	38	1	58	10	78		98		18	6	38	
19	1	39	1	59	10	79		99		19	12	39	
20	1	40	1	60	5	80		100		20	6	40	
மொத்தம் TOTAL	19	மொத்தம் TOTAL	19	மொத்தம் TOTAL	105	மொத்தம் TOTAL	46	மொத்தம் TOTAL		மொத்தம் TOTAL	167	மொத்தம் TOTAL	22

வினாவாரியாக ஒட்டு மொத்தம்
Question-wise Grand Total

189

பக்கவாரியாக ஒட்டு மொத்தம்
Page-wise Grand Total

189

AE: 504
GCP-388-30-57,00,000 Cps.-27-8-15 (HCL-6)-1SO: 12/4/16 PW
60:054CE: 12/4/16 FW
CE-054

தேர்வு எழுதுபவர் செய்யக்கூடியவை மற்றும் செய்யக்கூடாதவை
Do's & Dont's for Candidates

1. முகப்புச்சீட்டில் உரிய இடத்தில் கையொப்பமிட வேண்டும்.
Put your signature in the Top sheet in the appropriate place.
2. விடைத்தாளில் ஒரு பக்கத்திற்கு 20 முதல் 25 வரிகள் வரை எழுதவேண்டும்.
Write 20 to 25 lines in a page.
3. விடைத்தாளின் இருபுறத்திலும் எழுத வேண்டும்.
Write answers in both sides of paper.
4. செய்முறைகள் யாவும் விடைத்தாளின் பகுதியில் இடம் பெறவேண்டும்.
All rough works must be done on the lower part of the page.
5. சரியான வினா எண் தவறாமல் எழுத வேண்டும். வினா எண் எழுதாத மற்றும் தவறான வினா எண்கள் குறிப்பிடப்பட்டு எழுதப்பட்ட விடைகள் மதிப்பீடு செய்யப்படமாட்டாது.
Write the question numbers without fail. Answers without question numbers and wrong question numbers will not be valued.
6. இரு விடைகளுக்கிடையே இடைவெளி விட்டு எழுத வேண்டும்.
Leave space between two answers.
7. வினாத்தாளின் வரிசை (A or B) மதிப்பெண்கள் பக்கத்தில் குறிக்கப்படல் வேண்டும்.
Question paper booklet series (A or B) should be mentioned in the Marking Page
8. விடைத்தாளில் நீலம்/கருப்புமை கொண்ட பேனாவால் விடைகளை தெளிவாக எழுத வேண்டும்.
Answers must be legibly written either in Blue or Black ink pen.
9. விடைத்தாளில் எழுதாத பக்கங்களில் குறுக்குக்கோடு இடவேண்டும்.
Cross the unwritten pages.
1. வினாத்தாளில் எந்தவித குறியீடும் இடக் கூடாது.
No marking in the question paper.
2. விடைத்தாளை சேதப்படுத்தக் கூடாது.
Don't damage the answer paper.
3. விடைத்தாளில் எந்த ஒரு பக்கத்திலும் தேர்வு எண்/பெயர் எழுதக்கூடாது.
Don't write name, Register Number in any page of the answer book.
4. வண்ணக்கலர் கொண்ட பேனா/ பென்சில் எதையும் பயன்படுத்தக் கூடாது.
Don't write with sketch / colour pencils.
5. விடைத்தாள் கோட்டின் இடது ஓரத்தில் எழுதக்கூடாது.
Don't write on the left margin.
6. விடைத்தாள் புத்தகத்தின் எந்த தாளையும் கிழிக்கவோ/நீக்கவோ கூடாது.
Don't tare / remove any page from the answer book.



PART - A

1. 4) fourth quadrant ✓

2. 2) direct ✓

3. 4) b ✓

4. 1) -2 ✓

5. 1) $8\sqrt{5} \lambda$ ✓

6. 3) -1 ✓

7. 2) $\sqrt{2} - 1$ ✓

8. 1) 1 ✓

9. ~~4) $\frac{1}{10}$~~ 2) $\frac{1}{10}$ ✓

10. 4) $\frac{dy}{dx} + xy = e^x$ is a linear differential equation in y ✓

10

10

Qn.No.

11.

$$4) P \wedge (\sim P)$$

12.

$$1) 4.$$

13.

$$2) 2.$$

14.

$$1) 5$$

15.

$$2) z_1 + z_3 = z_2 + z_4$$

16.

$$4) z = 0.$$

17.

$$1) f(2a - z) = f(z)$$

18.

$$4) (ii), (iii), (iv)$$

19.

2) minimum value at $z = 0$.

20.

$$4) \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$



16

9

வினா எண்.
Qn.No.

21.

$$1) \tan^{-1} \left(\frac{2m}{m^2-1} \right)$$

22.

$$4) 5$$

23.

$$1) 0$$

24.

$$4) 3\sqrt{30}$$

25.

$$4) 8$$

(10)

26.

$$1) 4$$

27.

$$4) 9\pi$$

28.

2) only a semi-group.

29.

4) is a perpendicular bisector of the line joining z_1 and z_2

30.

$$3) 80.$$

19

ଶିକ୍ଷକ
ନାମ
Qn.No.

31. 2) trivial solution and infinitely many non-trivial solutions.

32. 2) $x < a$ and $x > -a$.

33. 4) $\frac{x \log x}{g(x)}$

34. 1) 4.

35. 1) $k^3 \det(A)$

36. 4) 200 m.

37. 3) (1, 1, 2)

38. 2) $x = \pm \frac{8}{\sqrt{5}}$

39. 3) $a = \frac{1}{|m|}$

40. 2) $\frac{d^2y}{dx^2} = 0$



9



Part - c.

56.

Given equations :

$$2x - 3y - 8z = -10.$$

$$3x + y - 4z = 0.$$

$$2x + 5y + 6z - 13 = 0 \Rightarrow 2x + 5y + 6z = 13$$

$$\begin{pmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{pmatrix}
 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 13 \end{pmatrix}$$

$A \quad \quad \quad x \quad \quad \quad B$

The augmented form

$$(A, B) = \left(\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{array} \right)$$

ආරම්භක ආකාරය
 ආරම්භක ආකාරය

$$\sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{pmatrix}$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

வினா
எண்.
Qn.No.

$$57. \quad \vec{a} = \vec{i} + \vec{j} + \vec{k} = (1, 1, 1)$$

$$10 \quad \vec{b} = 2\vec{i} + \vec{k} = (2, 0, 1)$$

$$\vec{c} = 2\vec{i} + \vec{j} + \vec{k} = (2, 1, 1)$$

$$\vec{d} = \vec{i} + \vec{j} + 2\vec{k} = (1, 1, 2)$$

TO PROVE: $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \vec{b} \vec{c}] \vec{d} - [\vec{a} \vec{b} \vec{d}] \vec{c}$

L.H.S.

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= \vec{i} + \vec{j} - 2\vec{k}$$

$$(\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \vec{i} - 3\vec{j} + \vec{k}$$

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{c} & \vec{d} & \vec{r} \\ 1 & 1 & -2 \\ 1 & -3 & 1 \end{vmatrix}$$

$$= -5\vec{i} - 3\vec{j} - 4\vec{k} \rightarrow \textcircled{1}$$

P.H.S

$$[\vec{a} \ \vec{b} \ \vec{d}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= -1 - 3 + 2 = -2$$

$$[\vec{a} \ \vec{b} \ \vec{d}] \vec{c} = -2 \{ 2\vec{i} + \vec{j} + \vec{k} \}$$

$$= -4\vec{i} - 2\vec{j} - 2\vec{k}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= -1 + 0 + 2 = 1$$

$$[\vec{a} \ \vec{b} \ \vec{c}] \vec{d} = 1 \{ \vec{i} + \vec{j} + 2\vec{k} \} = \vec{i} + \vec{j} + 2\vec{k}$$

வினா
எண்.
Qn.No.

$$\begin{aligned}
 [\vec{a} \vec{b} \vec{c}] \vec{d} - [\vec{a} \vec{b} \vec{d}] \vec{c} &= -4\vec{i} - 2\vec{j} - 2\vec{k} \\
 &\quad - (\vec{i} + \vec{j} + 2\vec{k}) \\
 &= -5\vec{i} - 3\vec{j} - 4\vec{k} \quad \longrightarrow \textcircled{2}
 \end{aligned}$$

By ① & ②

we have.

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}] \vec{c} - [\vec{a}, \vec{b}, \vec{c}] \vec{d}$$

58.

SP

Given line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$

dir $\vec{u} = (2, 3, -2)$

$\vec{b} = (2, 2, 1)$

Given point $\vec{a} = (-1, 1, -1)$

The Req plane passes through \vec{a}, \vec{b} and
 \parallel to \vec{u} .

Vector Equation $\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1-s)(-\vec{i} + \vec{j} - \vec{k}) + s(2\vec{i} + 2\vec{j} + \vec{k}) + t(2\vec{i} + 3\vec{j} - 2\vec{k})$$

Cart Equation :

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0$$

$$\begin{vmatrix} x+1 & y-1 & z+1 \\ -3 & -1 & -2 \\ 2 & 3 & -2 \end{vmatrix} = 0$$

$$(x+1)(2+6) - (y-1)(6+4) + (z+1)(-9+2) = 0$$

$$(x+1)(8) - (y-1)(10) + (z+1)(-7) = 0$$

$$8x + 8 - 10y + 10 - 7z - 7 = 0$$

$$8x - 10y - 7z + 11 = 0$$

59.

10

$$\operatorname{Re} \left(\frac{z+1}{z+i} \right) = 1$$

$$\text{Let } z = x + iy$$

Consider :

$$\frac{z+1}{z+i} = \frac{x+iy+1}{x+iy+i}$$

$$= \frac{(x+1) + iy}{x + i(y+1)}$$

$$= \frac{(x+1) + iy}{x + i(y+1)} \times \frac{x - i(y+1)}{x - i(y+1)}$$

$$\frac{z+1}{z+i} = \frac{(x+1)x + y(y+1) + \text{Imp part}}{x^2 + (y+1)^2}$$

Given :-

$$\operatorname{Re} \left(\frac{z+1}{z+i} \right) = 1$$

$$\frac{(x+1)x + y(y+1)}{x^2 + (y+1)^2} = 1$$

$$\cancel{x+x} + \cancel{y^2+y} = x^2 + y^2 + 2y + 1$$

$$\boxed{x - y = 1}$$

∴ the reqd locus which is a straight line.

60.

Given parabola:

$$y^2 + 8x - 6y + 1 = 0$$

$$y^2 - 6y = -8x - 1$$

$$(y-3)^2 - 9 = -8x - 1$$

$$(y-3)^2 = -8x + 8$$

$$(y-3)^2 = -8(x+1) \quad \alpha$$

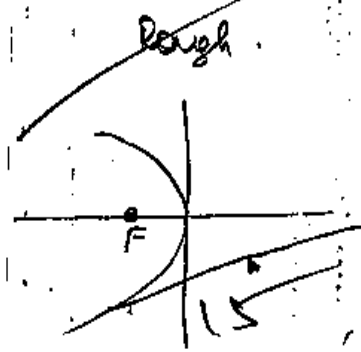
$$y^2 = -8x \quad \text{s.t.} \quad y = y-3$$

$$x = x+1 \quad \tau$$

$$\left\{ \text{S.F. } y^2 = -4ax \right\}$$

Comparing $-4a = -8$

$$\boxed{a = 2}$$



வினா எண்.
Qn.No.

	Went to x, y axis	Went to x, y axis
Vertex	$(0, 0)$	$x = x - 1$ $y = y + 3$ $x = -1$ $y = 3$
Focus	$(a, 0) = (2, 0)$	$(-1, 3)$ $x = -3$ $y = 3$ $(-3, 3)$
Eqn of direct	$x = a$ $x = 2$	$x = 1$
Latus rectum	$x = -a$ $x = -2$	$x = -3$
L.L.R.	$4a = 8$ units	8 units
Axis	$y = 0$	$y = 3$

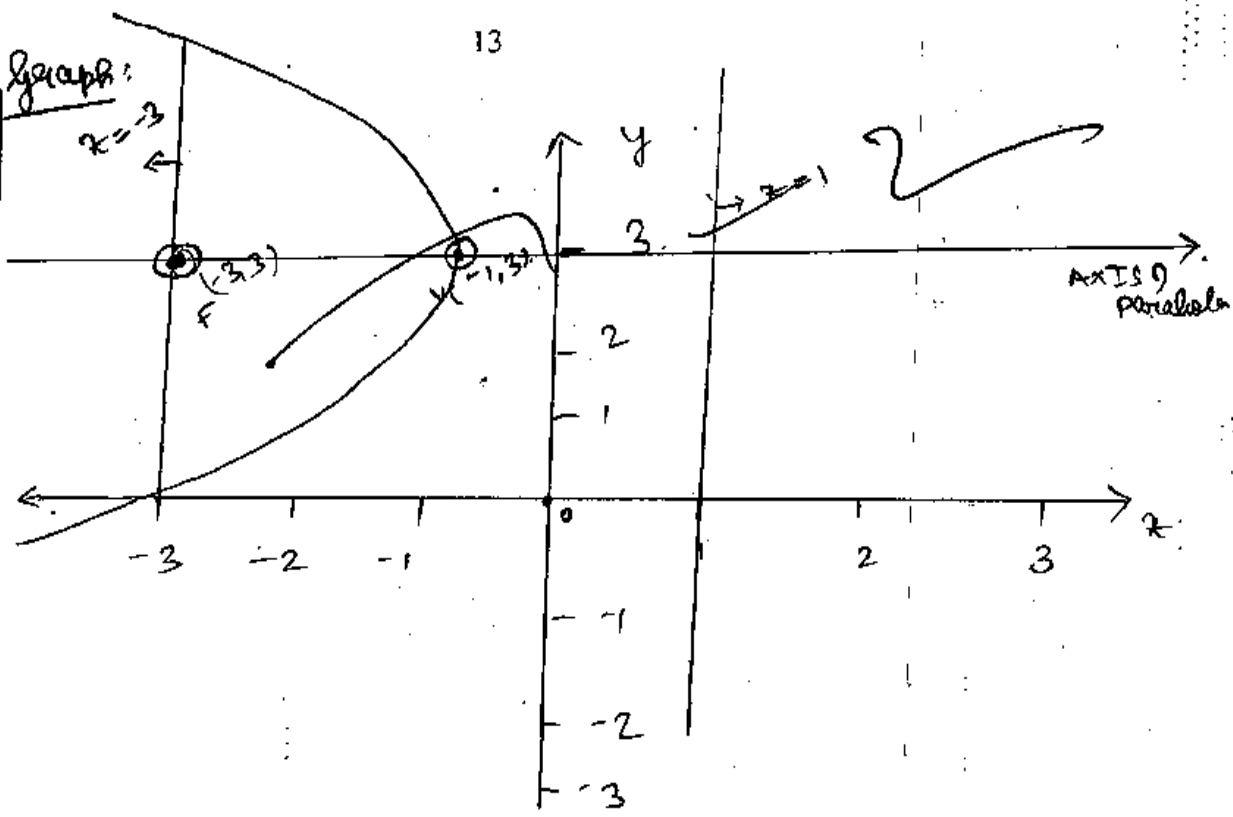
Result: Axis $y = 3$
Vertex $(-1, 3)$
Focus $(-3, 3)$

Eqn of dir $x = 1$
Latus rect $x = -3$
L.L.R = 8 units

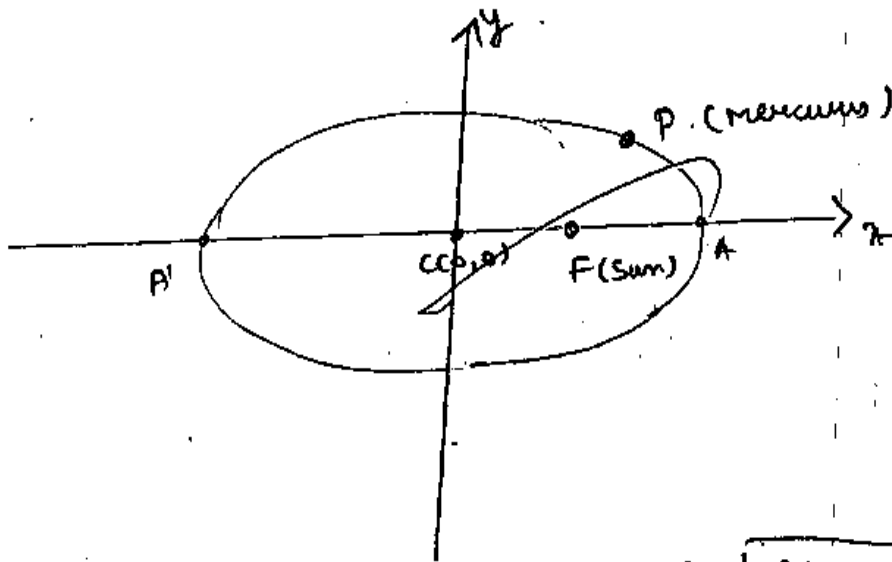


வினா எண்.
Qn.No.

Graph:



61.
10



Wkt $CA = a$

Given

$a = 36$ million miles

$e = 0.206$

To find

- i) $FA = ?$
- ii) $FA' = ?$

Handwritten scribbles and a large number '10' at the bottom right.

$$CA = CF + FA$$

$$FA = CA - CF$$

$$= a - ae$$

$$FA = a(1 - e)$$

$$= 36(1 - 0.206)$$

$$= 36(0.794)$$

$$\begin{array}{r} 9.9910 \\ \times 0.206 \\ \hline 0.794 \\ \hline \times 36 \\ \hline 4764 \\ 2382 \\ \hline 28.584 \end{array}$$

$$FA = 28.584 \text{ million miles}$$

$$FA' = CF + CA'$$

$$= a + ae$$

$$= a(1 + e)$$

$$= 36(1 + 0.206)$$

$$\begin{array}{r} 1.000 \\ + 0.206 \\ \hline 1.206 \\ \hline \times 36 \\ \hline 7236 \\ 3618 \\ \hline 43.416 \end{array}$$

$$FA' = 43.416 \text{ million miles}$$

Result:

i) Closest distance mercury come to Sun is 28.584 million miles

ii) The greatest possible distance between Mercury and Sun is 43.416 million miles

Given line $x - y + 4 = 0$

$$y = x + 4$$

S.F. $(y = mx + c)$

Comparing $\boxed{m=1}$
 $\boxed{c=4}$

Given ellipse $x^2 + 3y^2 = 12$

$$\frac{x^2}{12} + \frac{y^2}{4} = 1$$

S.F. $\left\{ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\}$

$\boxed{a^2=12}$ $\boxed{b^2=4}$

The condition for the line to be tg to ellipse

$$c^2 = a^2 m^2 + b^2$$

$$16 = 12(1) + 4$$

$$16 = 16 \quad \text{is true}$$

\Rightarrow The given line is the tg to the ellipse.

வினா
எண்.
Qn.No.

$$\begin{aligned} \text{Point of Contact} &= \left(-\frac{a^2 m}{c}, \frac{b^2}{c} \right) \\ &= \left(\frac{-12(1)}{4}, \frac{4}{4} \right) \\ &= (-3, 1) \end{aligned}$$

Result.

The point of contact is $(-3, 1)$.

64.

10

Given $u = \sin 3x \cos 4y$.

To verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

வினா
எண்.
Qn.No.
2.

L.H.S

$$\frac{\partial u}{\partial x} = 3 \cos 3x \cos 4y.$$

$$\frac{\partial^2 u}{\partial x \partial y} = -12 \cos 3x \sin 4y. \rightarrow \textcircled{1}$$

R.H.S

$$\frac{\partial u}{\partial y} = -4 \sin 3x \cos^2 4y$$

$$\frac{\partial^2 u}{\partial y \partial x} = -12 \cos 3x \sin^2 4y. \rightarrow \textcircled{2}$$

By $\textcircled{1}$ & $\textcircled{2}$ we have.

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$

65.

Given

$$x = a(t + \sin t)$$

$$y = a(1 + \cos t) \quad 0 \leq t \leq 2\pi.$$

$$S.A = \int_a^b y \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt.$$

Let us take the limits 0 to π .

$$a = 0 \quad b = \pi.$$

$$\frac{dx}{dt} = a(1 + \cos t)$$

$$\frac{dy}{dt} = a(-\sin t)$$

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 &= a^2(1 + \cos t)^2 \\ &= a^2(1 + 2\cos t + \cos^2 t) \end{aligned}$$

$$\left(\frac{dy}{dt}\right)^2 = a^2 \sin^2 t.$$

$$S.A = 2\pi \int_0^{\pi} y \sqrt{a^2 (1 + 2\cos t (\cos^2 t) + a^2 \sin^2 t) dt}$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2 (1 + 2\cos t + \cos^2 t + \sin^2 t) dt}$$

$\cos^2 t + \sin^2 t = 1$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2 (2 + 2\cos t) dt}$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2 (1 + \cos t) dt}$$

$$= 2\pi \int_0^{\pi} y \cdot \sqrt{a^2 2 \left(2 \cos^2 \frac{t}{2}\right) dt}$$

$$= 2\pi \int_0^{\pi} y \sqrt{a^2 4 \cos^2 \frac{t}{2} dt}$$

$$= 2\pi \int_0^{\pi} y [2a \cos \frac{t}{2}] dt$$

$$= 2\pi \int_0^{\pi} a(1 + \cos t) [2a \cos \frac{t}{2}] dt$$

$$= 4\pi a^2 \int_0^{\pi} \cos^2 \frac{t}{2} \cos \frac{t}{2} dt$$

$$= 4\pi a^2 \int_0^{\pi} \cos^3 \frac{t}{2} dt$$

Qn.No.

$$= 4\pi a^2 \int_0^{\pi/2} \cos^3 t/2 dt$$

$$\begin{aligned} t/2 &= x \\ \frac{dt}{dx} &= 2 \\ dt &= 2dx \end{aligned}$$

t	0	π
x	0	$\pi/2$

$$= 4\pi a^2 \int_0^{\pi/2} 2 \cos^3 x dx$$

$$= 8\pi a^2 \left[\frac{2}{3} \cdot 1 \right]$$

$$= \frac{16\pi a^2}{3} \text{ Sq. unit}$$

By Reduction Formula:
 $\int_0^{\pi/2} \cos^n x dx = \frac{n-1}{n} \frac{n-2}{n-3} \dots \frac{2}{3}$
 n is odd.

The Rqd Surface

$$\text{area} = \frac{16\pi a^2}{3}$$

63.

$$x = a(1 + \sin^2 \theta) \quad y = a(1 + \cos \theta)$$

$$\theta = \pi/2$$

$$\frac{dx}{d\theta} = a(1 + \cos \theta) = a \{ 2 \cos^2 \theta/2 \}$$

$$\frac{dy}{d\theta} = a(-\sin \theta) = -a \{ 2 \sin \theta/2 \cos \theta/2 \}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{a(-\sin \theta)}{a(1 + \cos \theta)}$$

$$= \frac{-a \{ 2 \sin \theta/2 \cos \theta/2 \}}{a \{ 2 \cos^2 \theta/2 \}}$$

$$= \frac{-\sin \theta/2}{\cos \theta/2}$$

$$\boxed{\frac{dy}{dx} = -\tan \theta/2} \Rightarrow m$$

தரப்பட்டது

at $\theta = \pi/2$

$$x = a(\pi/2 + 1)$$

தரப்பட்டது

$$y = a$$

The co-ordinate is (x, y) is $(a\pi/2 + a, a)$

Eqn of tg at $(a\pi/2 + a, a)$

$$(y - y_1) = m(x - x_1)$$

$$(y - a) = -\tan\theta_{1/2} (x - (a\pi/2 + a))$$

$$y - a = -\tan\theta_{1/2} (x - \frac{a\pi}{2} - a)$$

Slope of normal $m_2 = \frac{-1}{m_1} = +\cot\theta_2$

0123456789101112
 131415161718192021222324252627282930313233343536373839404142434445464748495051525354555657585960616263646566676869707172737475767778798081828384858687888990919293949596979899100101102103104105106107108109110111112113114115116117118119120121122123124125126127128129130131132133134135136137138139140141142143144145146147148149150151152153154155156157158159160161162163164165166167168169170171172173174175176177178179180181182183184185186187188189190191192193194195196197198199200201202203204205206207208209210211212213214215216217218219220221222223224225226227228229230231232233234235236237238239240241242243244245246247248249250251252253254255256257258259260261262263264265266267268269270271272273274275276277278279280281282283284285286287288289290291292293294295296297298299300301302303304305306307308309310311312313314315316317318319320321322323324325326327328329330331332333334335336337338339340341342343344345346347348349350351352353354355356357358359360361362363364365366367368369370371372373374375376377378379380381382383384385386387388389390391392393394395396397398399400401402403404405406407408409410411412413414415416417418419420421422423424425426427428429430431432433434435436437438439440441442443444445446447448449450451452453454455456457458459460461462463464465466467468469470471472473474475476477478479480481482483484485486487488489490491492493494495496497498499500501502503504505506507508509510511512513514515516517518519520521522523524525526527528529530531532533534535536537538539540541542543544545546547548549550551552553554555556557558559560561562563564565566567568569570571572573574575576577578579580581582583584585586587588589590591592593594595596597598599600601602603604605606607608609610611612613614615616617618619620621622623624625626627628629630631632633634635636637638639640641642643644645646647648649650651652653654655656657658659660661662663664665666667668669670671672673674675676677678679680681682683684685686687688689690691692693694695696697698699700701702703704705706707708709710711712713714715716717718719720721722723724725726727728729730731732733734735736737738739740741742743744745746747748749750751752753754755756757758759760761762763764765766767768769770771772773774775776777778779780781782783784785786787788789790791792793794795796797798799800801802803804805806807808809810811812813814815816817818819820821822823824825826827828829830831832833834835836837838839840841842843844845846847848849850851852853854855856857858859860861862863864865866867868869870871872873874875876877878879880881882883884885886887888889890891892893894895896897898899900901902903904905906907908909910911912913914915916917918919920921922923924925926927928929930931932933934935936937938939940941942943944945946947948949950951952953954955956957958959960961962963964965966967968969970971972973974975976977978979980981982983984985986987988989990991992993994995996997998999100010011002100310041005100610071008100910101011101210131014101510161017101810191020102110221023102410251026102710281029103010311032103310341035103610371038103910401041104210431044104510461047104810491050105110521053105410551056105710581059106010611062106310641065106610671068106910701071107210731074107510761077107810791080108110821083108410851086108710881089109010911092109310941095109610971098109911001101110211031104110511061107110811091110111111121113111411151116111711181119112011211122112311241125112611271128112911301131113211331134113511361137113811391140114111421143114411451146114711481149115011511152115311541155115611571158115911601161116211631164116511661167116811691170117111721173117411751176117711781179118011811182118311841185118611871188118911901191119211931194119511961197119811992002012022032042052062072082092102112122132142152162172182192202212222232242252262272282292302312322332342352362372382392402412422432442452462472482492502512522532542552562572582592602612622632642652662672682692702712722732742752762772782792802812822832842852862872882892902912922932942952962972982993003013023033043053063073083093103113123133143153163173183193203213223233243253263273283293303313323333343353363373383393403413423433443453463473483493503513523533543553563573583593603613623633643653663673683693703713723733743753763773783793803813823833843853863873883893903913923933943953963973983994004014024034044054064074084094104114124134144154164174184194204214224234244254264274284294304314324334344354364374384394404414424434444454464474484494504514524534544554564574584594604614624634644654664674684694704714724734744754764774784794804814824834844854864874884894904914924934944954964974984995005015025035045055065075085095105115125135145155165175185195205215225235245255265275285295305315325335345355365375385395405415425435445455465475485495505515525535545555565575585595605615625635645655665675685695705715725735745755765775785795805815825835845855865875885895905915925935945955965975985996006016026036046056066076086096106116126136146156166176186196206216226236246256266276286296306316326336346356366376386396406416426436446456466476486496506516526536546556566576586596606616626636646656666676686696706716726736746756766776786796806816826836846856866876886896906916926936946956966976986997007017027037047057067077087097107117127137147157167177187197207217227237247257267277287297307317327337347357367377387397407417427437447457467477487497507517527537547557567577587597607617627637647657667677687697707717727737747757767777787797807817827837847857867877887897907917927937947957967977987998008018028038048058068078088098108118128138148158168178188198208218228238248258268278288298308318328338348358368378388398408418428438448458468478488498508518528538548558568578588598608618628638648658668678688698708718728738748758768778788798808818828838848858868878888898908918928938948958968978988999009019029039049059069079089099109119129139149159169179189199209219229239249259269279289299309319329339349359369379389399409419429439449459469479489499509519529539549559569579589599609619629639649659669679689699709719729739749759769779789799809819829839849859869879889899909919929939949959969979989991000100110021003100410051006100710081009101010111012101310141015101610171018101910201021102210231024102510261027102810291030103110321033103410351036103710381039104010411042104310441045104610471048104910501051105210531054105510561057105810591060106110621063106410651066106710681069107010711072107310741075107610771078107910801081108210831084108510861087108810891090109110921093109410951096109710981099110011011102110311041105110611071108110911101111111211131114111511161117111811191120112111221123112411251126112711281129113011311132113311341135113611371138113911401141114211431144114511461147114811491150115111521153115411551156115711581159116011611162116311641165116611671168116911701171117211731174117511761177117811791180118111821183118411851186118711881189119011911192119311941195119611971198119920020120220320420520620720820921021121221321421521621721821922022122222322422522622722822923023123223323423523623723823924024124224324424524624724824925025125225325425525625725825926026126226326426526626726826927027127227327427527627727827928028128228328428528628728828929029129229329429529629729829930030130230330430530630730830931031131231331431531631731831932032132232332432532632732832933033133233333433533633733833934034134234334434534634734834935035135235335435535635735835936036136236336436536636736836937037137237337437537637737837938038138238338438538638738838939039139239339439539639739839940040140240340440540640740840941041141241341441541641741841942042142242342442542642742842943043143243343443543643743843944044144244344444544644744844945045145245345445545645745845946046146246346446546646746846947047147247347447547647747847948048148248348448548648748848949049149249349449549649749849950050150250350450550650750850951051151251351451551651751851952052152252352452552652752852953053153253353453553653753853954054154254354454554654754854955055155255355455555655755855956056156256356456556656756856957057157257357457557657757857958058158258358458558658758858959059159259359459559659759859960060160260360460560660760860961061161261361461561661761861962062162262362462562662762862963063163263363463563663763863964064164264364464564664764864965065165265365465565665765865966066166266366466566666766866967067167267367467567667767867968068168268368468568668768868969069169269369469569669769869970070170270370470570670770870971071171271371471571671771871972072172272372472572672772872973073173273373473573673773873974074174274374474574674774874975075175275375475575675775875976076176276376476576676776876977077177277377477577677777877978078178278378478578678778878979079179279379479579679779879980080180280380480580680780880981081181281381481581681781881982082182282382482582682782882983083183283383483583683783883984084184284384484584684784884985085185285385485585685785885986086186286386486586686786886987087187287387487587687787887988088188288388488588688788888989089189289389489589689789889990090190290390490590690790890991091191291391491591691791891992092192292392492592692792892993093193293393493593693793893994094194294394494594694794894995095195295395495595695795895996096196296396496596696796896997097197297397497597697797897998098198298398498598698798898999099199299399499599699799899910001001100210031004100510061007100810091010101110121013101410151016101710181019102010211022102310241025102610271028102910301031103210331034103510361037103810391040104110421043104410451046104710481049105010511052105310541055105610571058105910601061106210631064106510661067106810691070107110721073107410751076107710781079108010811082108310841085108610871088108910901091109210931094109510961097109810991100110111021103110411051106110711081109111011111112111311141115111611171118111911201121112211231124112511261127112811291130113111321133113411351136113711381139114011411142114311441145114611471148114911501151115211531154115511561157115811591160116111621163116411651166116711681169117011711172117311741175117611771178117911801181118211831184118511861187118811891190119111921193119411951196119711981199200201202203204205206207208209210211212213214215216217218219220221222223224225226227228229230231232233234235236237238239240241242243244245246247248249250251252253254255256257258259260261262263264265266267268269270271272273274275276277278279280281282283284285286287288289290291292293294295296297298299300301302303304305306307308309310311312313314315316317318319320321322323324325326327328329330331332333334335336337338339340341342343344345346347348349350351352353354355356357358359360361362363364365366367368369370371372373374375376377378379380381382383384385386387388389390391392393394395396397398399400401402403404405406407408409410411412413414415416417418419420421422423424425426427428429430431432433434435436437438439440441442443444445446447448449450451452453454455456457458459460461462463464465466467468469470471472473474475476477478479480481482483484485486487488489490491492493494495496497498499500501502503504505506507508509510511512513514515516517518519520521522523524525526527528529530531532533534535536537538539540541542543544545546547548549550551552553554555556557558559560561562563564565566567568569570571572573574575576577578579580581582583584585586587588589590591592593594595596597598599600601602603604605606607608609610611612613614615616617618619620621622623624625626627628629630631632633634635636637638639640641642643644645646647648649650651652653654655656657658659660661662663664665666667668669670671672673674675676677678679680681682683684685686687688689690691692693694695696697698699700701702703704705706707708709710711712713714715716717718719720721722723724725726727728729730731732733734735736737738739740741742743744745746747748749750751752753754755756757758759760761762763764765766767768769770771772773774775776777778779780781782783784785786787788789790791792793794795796797798799800801802803804805



To
10

a)

Given curve $f(x) y = e^{-x^2}$

$$y' = e^{-x^2} \cdot (-2x)$$

$$y = e^{-x^2}$$

$$-2y + -2x y'$$

$$-2e^{-x^2} + 4x^2 e^{-x^2}$$

$$y'' = -2e^{-x^2} + 4x^2 e^{-x^2}$$

Put $y'' = 0$

$$-2e^{-x^2} + 4x^2 e^{-x^2} = 0$$

$$+ 2x^2 e^{-x^2} = + \frac{2}{1} x^2 e^{-x^2}$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{1}{\sqrt{2}}$$



$$y = e^{-\frac{1}{2}}$$

10

வினா
எண்.
Qn.No.

Interval	Sign of y''	Concavity
$-\alpha < x < -\frac{1}{\sqrt{2}}$ Say (-1) .	+ve	Concave upward.
$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ Say $x=0$	-ve	Concave downward.
$\frac{1}{\sqrt{2}} < x < \alpha$ Say $x=$	+ve	Concave upward.

① The sign of y'' changes from +ve to -ve at $x = -\frac{1}{\sqrt{2}}$,

$\left(-\frac{1}{\sqrt{2}}, y\left(-\frac{1}{\sqrt{2}}\right)\right)$ is the ~~Point~~ point of inflection

$\left(-\frac{1}{\sqrt{2}}, e^{-\frac{1}{\sqrt{2}}}\right)$ is the point of inflection

② The sign of f'' change from $-ve$ to $+ve$ at $x = \frac{1}{\sqrt{2}}$.

$(\frac{1}{\sqrt{2}}, f(x))$ is the pt of inflection

$(\frac{1}{\sqrt{2}}, e^{-1/2})$ is the pt of inflection

Result :

1. $(-\infty < x < -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}} < x < \infty) \rightarrow$ concave upwards

2. $(-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}) \rightarrow$ concave downward

3. $(-\frac{1}{\sqrt{2}}, e^{-1/2})$ is the pt of inflection.

41.

Given Equation

$$x + y + 2z = 4$$

$$2x + 2y + 4z = 8$$

$$3x + 3y + 6z = 10$$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{vmatrix} = 0 \quad \{ R_1 \equiv R_2 \}$$

\Rightarrow Solution may
or may not
exist

$$\Delta_x = \begin{vmatrix} 4 & 1 & 2 \\ 8 & 2 & 4 \\ 10 & 3 & 6 \end{vmatrix} = 0 \quad \{ R_2 \propto R_3 \}$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ 3 & 10 & 6 \end{vmatrix} = 0 \quad \{ R_1 \propto R_3 \}$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 2 & 8 \\ 3 & 3 & 10 \end{vmatrix} = 0 \quad \{ R_1 = R_2 \}$$

$$\Delta = 0 \quad \text{and} \quad \Delta_x = \Delta_y = \Delta_z = 0$$

\Rightarrow Consistent and infinitely many solutions

Let consider 2×2 minor of Δ

It is found that all the 2×2 minor of Δ has $\Delta = 0$

But if one of the 2×2 minor of $\Delta_x \neq 0$

$$\begin{vmatrix} 8 & 2 \\ 10 & 3 \end{vmatrix} = 24 - 20 = 4 \neq 0$$

Since, on taking 2×2 minor, we have $\Delta = 0$ But $\Delta_x \neq 0$.

\Rightarrow no solution exist

ଶିକ୍ଷା
କ୍ରମ.
Qn. No.

42. Let $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ be a 3×3 matrix

$$A_c = \begin{pmatrix} +(-4) & +(-1) & +4 \\ -(+3) & +0 & -(-4) \\ +(-3) & -(-1) & +3 \end{pmatrix}$$

Co-factor of -4 is $\begin{vmatrix} 0 & 1 \\ 4 & 3 \end{vmatrix} = -4$

Co-factor of -3 is $\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = -1$

Co-factor of -3 is $\begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4$

Co-factor of 1 is $\begin{vmatrix} -3 & -3 \\ 4 & 3 \end{vmatrix} = +3$

Cofactor of 0 is $\begin{vmatrix} -4 & -3 \\ 4 & 3 \end{vmatrix}$

$$= -12 + 12 = 0$$

Cofactor of 1 is $\begin{vmatrix} -4 & -3 \\ 4 & 4 \end{vmatrix}$

$$= -16 + 12$$

$$= -4$$

Cofactor of 4 is $\begin{vmatrix} -3 & -3 \\ 0 & 1 \end{vmatrix}$

$$= -3$$

Cofactor of 4 is $\begin{vmatrix} -4 & -3 \\ 1 & 1 \end{vmatrix}$

$$= -4 + 3 = -1$$

Cofactor of 3 is $\begin{vmatrix} -4 & -3 \\ 1 & 0 \end{vmatrix}$ = +3

வினா
எண்.
Qn.No.

$$A_C = \begin{pmatrix} -4 & +1 & 4 \\ -3 & 0 & 4 \\ -3 & +1 & 3 \end{pmatrix}$$

$$A_C^T = \begin{pmatrix} 4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{pmatrix} \rightarrow \textcircled{2}$$

$$= A \text{ itself} \therefore \rightarrow$$

Therefore By $\textcircled{1}$ & $\textcircled{2}$ we have

adjoint A of the given matrix
is A itself.

44

i)

$$\vec{F} = (2, -2, 1)$$

$$5\vec{F} = \frac{5(2, -2, 1)}{\sqrt{4+4+1}}$$

$$= \frac{5(2, -2, 1)}{\sqrt{9}}$$

$$= \frac{5}{3}(2, -2, 1)$$

d = to- From.

$$= (5, 3, 7) - (1, 2, 3)$$

$$d = (4, 1, 4)$$

$$W = \vec{F} \cdot \vec{d}$$

$$= \frac{5}{3}(2, -2, 1) \cdot (4, 1, 4)$$

$$= \frac{5}{3}(8 - 2 + 4)$$

$$= \frac{5}{3}(10)$$

$$= \frac{50}{3} \text{ units}$$

6

ii). $A = (-1, 4, -3)$.

Sphere: $x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0$.

G. F. $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

On Comparing .

$$2u = -3$$

$$\boxed{u = -\frac{3}{2}}$$

$$2u = -2$$

$$\boxed{u = -1}$$

$$2w = 2$$

$$\boxed{w = 1}$$

$$\boxed{d = -15}$$

$$\text{Centre} = (-u, -v, -w)$$

$$= \left(\frac{3}{2}, 1, -1\right)$$

By Mid Point Formula. Let (x, y, z) be the point B)

$$\left(\frac{-1+x}{2}, \frac{4+y}{2}, \frac{-3+z}{2}\right) = \left(\frac{3}{2}, 1, -1\right)$$

Comparing

$$\frac{-1+z}{z} = \frac{3}{2}$$

$$\boxed{z=4}$$

$$\frac{4+y}{2} = 1$$

$$4+y = 2$$

$$\boxed{y=-2}$$

$$\frac{-3+z}{2} = -1$$

$$-3+z = -2$$

$$\boxed{z=+1}$$

The reqd point B is $(4, -2, +1)$.

45.

b

Given equation $x^4 - 4x^2 + 8x + 35 = 0$

$$\text{Given root} = 2 + \sqrt{3}i$$

$$\text{Other root} = 2 - \sqrt{3}i$$

{ Complex root
occur in
conjugate
pairs }

$$\text{Sum of Roots} = 4$$

$$\text{Product of Roots} = 2^2 + (\sqrt{3})^2 = 4 + 3 = 7$$

$$\text{Corresponding factor} = x^2 - 4x + 7$$

To find the other factor

$$(x^4 - 4x^2 + 8x + 35) = (x^2 - 4x + 7)(x^2 + px + 5)$$

On comparing x

$$8 = 7p - 20$$

$$7p = 28$$

$$\boxed{p=4}$$

வினா
எண்.
Qn.No.

710) Find other roots.

$$x^2 + 4x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Here } a=1$$

$$b=4$$

$$c=5$$

4.1.

$$x = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

$$x = \frac{-4 \pm i2}{2}$$

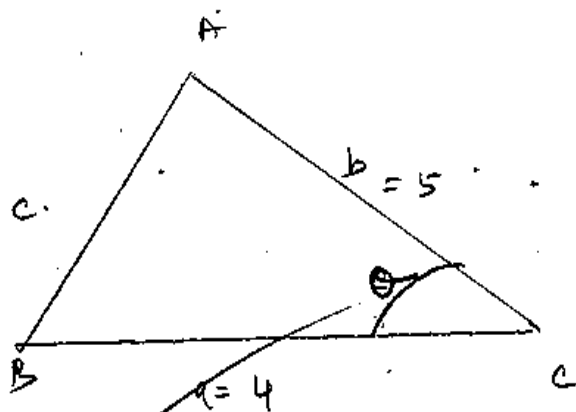
$$x = \frac{2(\sqrt{2 \pm i})}{2}$$

$$x = -2 \pm i$$

The roots are

$$2 \pm \sqrt{3}i, \quad -2 \pm i$$

47.



Given : $a = 4 \text{ m}$

$b = 5 \text{ m}$

$$\frac{d\theta}{dt} = 0.06 \text{ rad/sec}$$

To find $\frac{dA}{dt}$] $\theta = \pi/3$ = ?

$$\text{Area} = \frac{1}{2} ab \sin \theta$$

$$\frac{dA}{dt} = \frac{1}{2} ab \cos \theta \cdot \frac{d\theta}{dt}$$

$$= \frac{1}{2} \times 4 \times 5 \times \cos \pi/3 \times 0.06$$

$$= 10 \times \frac{1}{2} \times 0.03$$

$$= 0.3 \text{ m}^2/\text{sec}$$

The area of the triangle is increasing with rate of $0.3 \text{ m}^2/\text{sec}$.

46.

To prove $\tan^{-1}(x) < x \quad \forall x > 0$

$$f(x) = x - \tan^{-1}(x)$$

$$f'(x) = 1 - \frac{1}{1+x^2} > 0 \quad \forall x > 0$$

$f'(x)$ is strictly increasing $\forall x > 0$

Wkt. If $x > 0$.

$$f(x) > f(0)$$

$$x - \tan^{-1}(x) > 0 - \tan^{-1}(0)$$

$$x - \tan^{-1}(x) > 0$$

$$x > \tan^{-1} x$$

$$\tan^{-1} x < x$$

$$\forall x > 0$$

Hence proved.

Given $v = z e^{ax+by} \rightarrow (1)$

Given z is a homogeneous function of degree n in x & y .

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = n z \rightarrow (2)$$

On partially diff (1) wrt x .

$$\frac{\partial v}{\partial x} = z e^{ax+by} a$$

$$\frac{\partial v}{\partial x} = z e^{ax+by} a + e^{ax+by} \frac{\partial z}{\partial x}$$

$$x \frac{\partial v}{\partial x} = ax z e^{ax+by} + x e^{ax+by} \frac{\partial z}{\partial x}$$

On partial diff (1) wrt y .

$$\frac{\partial v}{\partial y} = z e^{ax+by} b + e^{ax+by} \frac{\partial z}{\partial y}$$

$$y \frac{\partial v}{\partial y} = by z e^{ax+by} + y e^{ax+by} \frac{\partial z}{\partial y}$$



$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} =$$

$$= e^{ax+by} \left[axz + x \frac{\partial z}{\partial x} \right] +$$

$$e^{ax+by} \left[byz + y \frac{\partial z}{\partial y} \right]$$

$$= e^{ax+by} \left[axz + byz + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} \right]$$

$$= e^{ax+by} \left[axz + byz + nz \right] \quad \text{by (2)}$$

$$= z e^{ax+by} [ax + by + n]$$

$$= v (ax + by + n)$$

*
Hence proved.



அரசுத் தேர்வுகள்
கருதல் விடைத்தரன்

51,
b

TO PROVE: $[(\sim q) \wedge p] \wedge q$ is a contradiction

P	q	$(\sim q)$	$[(\sim q) \wedge p]$	$[(\sim q) \wedge p] \wedge q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

Since all values of last column are F,
From ① it is clear that
 $[(\sim q) \wedge p] \wedge q$ is a contradiction

b



53.

$$n = 2$$

54.

$$p = \frac{20}{100} =$$

54.

$$n = 10$$

X be a r.v denoting choosing defective hel

$$P(\text{of choosing defective}) = \frac{20}{100} = \frac{1}{5} = p$$

$$P(\text{of choosing non defective}) = \frac{4}{5} = q$$

B. Exp.

$$P(X=r) = {}^n C_r p^r q^{n-r}$$

$$P(X=2) = {}^{10} C_2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^8$$

$$= \frac{10 \cdot 8}{1 \cdot 2} \frac{16}{5^2} \left(\frac{1}{5^8}\right)$$

$$= \frac{640}{5^{10}}$$

செய்தலின் விடிவு

Poisson:

$$P(X=2)$$

$$np = 10 \times \frac{1}{5} = 2 = \lambda$$

$$P(X=2) = \frac{e^{-2} 2^2}{2!}$$
$$= \frac{0.1353 \times 4}{1 \cdot 2}$$
$$= 0.2706$$

$$\frac{e^{-\lambda} \lambda^x}{x!}$$

$$\frac{0.1353 \times 2}{1}$$
$$= 0.2706$$

Result:

$$\text{Binomial} = \frac{45 \times 4^8}{5^{10}}$$

$$\text{Poisson} = 0.2706$$

அரசுத் தேர்வுகள்
கூடுதல் விடைத்தாள்

55

b

b) (i) Given $\left(\frac{1+i}{1-i}\right)^n = 1.$

Consider: $= \frac{1+i}{1-i}$

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{2}$$

$$= \frac{1 + 2i - 1}{2}$$

$$= \frac{2i}{2}$$

$$\left(\frac{1+i}{1-i}\right)^n = i^n = 1$$

$n = 4, 8, 12, \dots$
 $i^n = 1$

Least positive integer $n = 4$

$$(ii) \quad (i)^{1/3}$$

$$\text{consider } i^{\circ} : \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}.$$

$$(i)^{1/3} = \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)^{1/3}$$

$$= \left[\cos \left(2k\pi + \frac{\pi}{2} \right) + i \sin \left(2k\pi + \frac{\pi}{2} \right) \right]^{1/3}$$

$$k = 0, 1, 2$$

By De Moivre

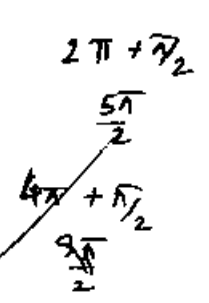
$$= \left[\cos \frac{1}{3} \left(2k\pi + \frac{\pi}{2} \right) + i \sin \frac{1}{3} \left(2k\pi + \frac{\pi}{2} \right) \right]$$

$$\text{when } k=0. \Rightarrow \cos \frac{\pi}{6}.$$

$$k=1 \Rightarrow \cos \frac{5\pi}{6}.$$

$$k=2 \Rightarrow \cos \frac{9\pi}{6}.$$

The values are $\cos \frac{\pi}{6}$, $\cos \frac{5\pi}{6}$, $\cos \frac{9\pi}{6}$





53.

$$n = 2$$

x be r.v denoting getting Aces.

$$P(S) = P(\text{Probability of getting Aces}) = \frac{4}{52}$$

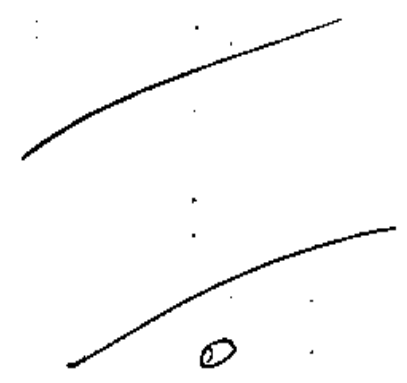
$$P(F) = \frac{48}{52}$$

$$\begin{aligned}
 P(x=0) &= P(F F) \\
 &= \frac{\overset{12}{48}}{\underset{13}{52}} \cdot \frac{\overset{12}{48}}{\underset{13}{52}} \\
 &= \frac{144}{169}
 \end{aligned}$$

$$\begin{aligned}
 P(x=1) &= P(SF \text{ or } FS) \\
 &= P(SF) \times 2 \\
 &= \frac{\overset{12}{48}}{\underset{13}{52}} \cdot \frac{1}{\underset{13}{52}} \times 2 \\
 &= \frac{24}{169}
 \end{aligned}$$

$$\begin{aligned}
 P(x=2) &= P(SS) \\
 &= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}
 \end{aligned}$$

Handwritten notes: P(SF or FS) and P(SF) x 2



Probability mass function



x	0	1	2	
$P(x=x)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$	$\sum p_i = 1$

$$\text{Mean} = \sum p_i x_i$$

$$= \frac{144}{169} (0) + \frac{24}{169} x(1) + \frac{2 \times 1}{169}$$

$$= \frac{24}{169} + \frac{2}{169}$$

$$E(x) = \frac{26}{169}$$

$$\text{Var} = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum p x^2$$

$$= (0)^2 \times \frac{144}{169} + (1^2) \times \frac{24}{169} + (2^2) \times \frac{1}{169}$$

$$= \frac{24}{169} + \frac{4}{169} = \frac{28}{169}$$

$$\text{Var} = \frac{28}{169} - \left(\frac{26}{169} \right)^2$$

$$= \frac{28}{169} - \frac{4}{169} = \frac{24}{169}$$



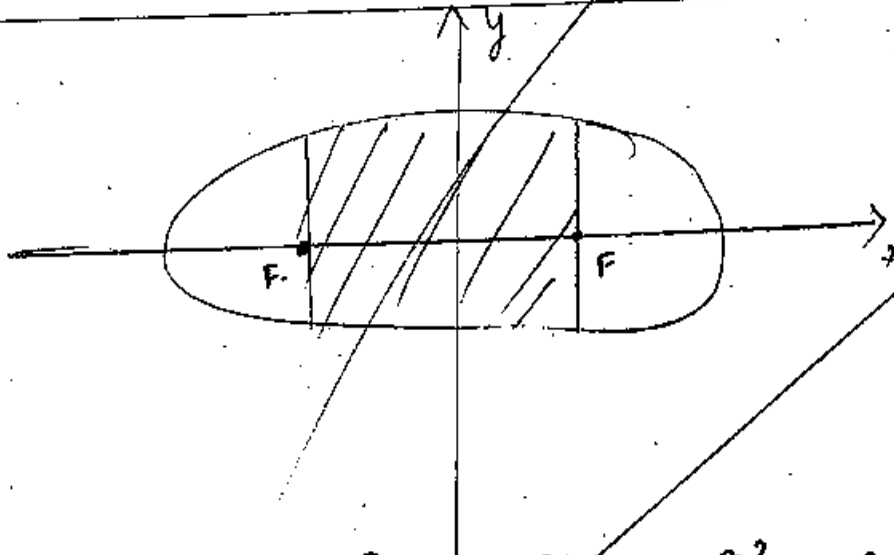
அரசுத் தேர்வுகள்
கட்டுநல் விடைத்தாள்

Result:

$$\text{Mean} = \frac{26}{169}$$

$$\text{Variance} = \frac{24}{169}$$

66.



The given ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$. (Sym about both the Axes)
↳ ①.

சமச்சுமையான
எலிப்சு

(PTO)
0

56.

$$x - 3y - 8z = -10$$

$$3x + y - 4z = 0$$

$$2x + 5y + 6z = 13$$

$$\begin{pmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 13 \end{pmatrix}$$

Augment form.

$$(A, B) = \begin{pmatrix} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -3 & -8 & -10 \\ 0 & +10 & 20 & +30 \\ 0 & 11 & 22 & 33 \end{pmatrix} \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -3 & -8 & -10 & \\ 0 & 10 & 20 & 30 & \\ 0 & \triangle & 0 & 0 & \end{array} \right) R_3 \rightarrow \frac{R_3}{11} - \frac{R_2}{10}$$

It is in echelon form.

$$P(A, B) = 2 = P(A) \quad \text{No of unknowns} \quad (3)$$

Consistent & has unique solutions.

To find solution, let us take $z = k$

$(k \in \mathbb{R})$ k is an arbitrary constant

By backward calculation.

~~$$10y + 20z = 30$$~~

$$10y + 20z = 30$$

$$10y = 30 - 20k$$

$$y = 3 - 2k$$

~~$$x - 3y - 8z = -10$$~~

$$x - 3y = -10 + 8k$$

10

$$z - 3(3 - 2k) = -10 + 8k.$$

$$z - 9 + 6k = -10 + 8k$$

$$\cancel{z} = \cancel{1} + \cancel{14k} \quad z = 2k - 1$$

$$\boxed{\cancel{z} = \cancel{14k - 1}}$$

The solution :

$$\cancel{z} = \cancel{14k - 1} \quad \cancel{z} = \cancel{2k - 1} \quad z = 2k - 1$$

$$y = 3 - 2k$$

$$z = k \quad k \in \mathbb{R}.$$

The solution are

$$z = 2k - 1$$

$$y = 3 - 2k$$

$$z = k \quad \{k \in \mathbb{R}\}$$

